Deep learning and the dynamical low-rank approximation

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The dynamical low-rank approximation [4, 6] is a reduced-order modeling technique that evolves a low-rank matrix or tensor approximation of a PDE or data-driven dynamics. It may be applied to high-dimensional deterministic problems where a fine mesh or large domain is required, or to stochastic problems, where a reduced-order Monte Carlo system is evolved, enabling inference on non-Gaussian distributions for uncertainty quantification. Recently, rank-adaptive integration schemes have been developed [5, 2, 1]. However, the proposed criteria for when and how to augment or decrease the rank of the solution are either computationally costly or heuristic and greedy; they usually only evaluate the system its current state in a Markov sense without regard for past or future states. Furthermore, the rank of the system is highly sensitive to the system's parameterization, particularly the choice of coordinates, which can be relatively arbitrary. We investigate deep learning techniques such as reinforcement learning and neural delayed equations [3] as alternative approaches to these heuristics choices. In learning how to better augment and parameterize our system, we develop schemes that adapt the subspace as we integrate in time, which better captures the dynamic, high-order curvature of the low-rank manifold. Because we combine techniques from numerical mathematics and machine learning, we preserve convergence guarantees and interpretability while improving accuracy and computational efficiency. We demonstrate this methodology on test cases in computational ocean acoustics.

References

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