Model Reduction of Navier-Stokes Equations using the Loewner Framework

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The Loewner framework is extended to compute reduced order models (ROMs) for systems governed by the incompressible Navier-Stokes (NS) equations. For quadratic ordinary differential equations (ODEs) it constructs a ROM directly from measurements of transfer function components derived from an expansion of the system's input-to-output map. Given measurements, no explicit access to the system is required to construct the ROM. To extend the Loewner framework, the NS equations are transformed into ODEs by projecting onto the subspace defined by the incompressibility condition, as in e.g., [1, 3, 5]. This projection is used theoretically, but avoided computationally.
A number of extensions over [1, 5] are presented. Specifically, system outputs involving the pressure lead to quadratic terms in the output of the projected system. These quadratic terms are systematically included. In addition, a number of computational issues are addressed that allow implementation of the approach using the trilinear form for the Navier-Stokes equations, rather than expressing it using the Kronecker product typically used to derive the ROM.
The straight forward application of the Loewner approach to the incompressible NS equations can lead to unstable ROMs [5]. This was already observed for Burgers' equation in [2] and in [4] for a related interpolation based ROM. Following [2, 5] a possible approach, which currently requires access to the NS system, to deal with these instabilities is outlined, and studied for specific systems.

References

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