Combining adaptive model order reduction and stochastic collocation for uncertainty quantification of vibroacoustic systems

U. Römer[1], M. Bollhöfer[2], H. K. Sreekumar[3], C. Blech[4], and S. C. Langer[3]

[1]*Institut für Dynamik und Schwingungen, Technische Universität Braunschweig, Germany*
[2]*Institut für Numerische Mathematik, Technische Universität Braunschweig, Germany*
[2]*Institut für Akustik, Technische Universität Braunschweig, Germany*

We address vibroacoustic problems, which are simulated with the finite element method and where material properties exhibit uncertainties. Modeling all uncertain parameters with a jointly distributed random vector leads to a high-dimensional reformulation of the problem, where both frequency and random material parameters need to be considered. In realistic applications, multi-query simulations are hardly possible due to the size of the system and the size of the parameter space. To overcome this problem, we propose to combine model order reduction (MOR) and stochastic collocation in a fully adaptive way [2].
The model problem reads

$$(-\omega^2 M(p) + i\omega D(p) + K(p))x(\omega, p) = F, \tag{1}$$

where $p \in \Gamma \subset \mathbb{R}^P$ denotes a parameter vector with an underlying probability distribution, ω the angular frequency and $M(p), D(p), K(p) \in \mathbb{C}^{n\times n}$. We are interested in computing moments of an output quantity, i.e. the quantity of interest

$$\mathbb{E}_p[y(\omega, \cdot)] = \mathbb{E}_p[\Psi(x(\omega, \cdot))], \tag{2}$$

for all $\omega \in [\underline{\omega}, \overline{\omega}]$, with a manageable computational effort. To that end, we apply a dimension-adaptive stochastic collocation method with Leja nodes. At each collocation node $\{p^{(i)}\}_{i=1}^{I}$ we build a reduced order model with the rational Arnoldi method and an adaptive selection of frequency expansion points $\{\omega_i^{(j)}\}_{j=1}^{J_i}$. The parametric output quantity is then obtained by interpolating the frequency response.
A major ingredient is the selection of adaptive expansion and collocation points, which we base on dual error indicators. In particular, we require the adjoint of both the full and reduced order model to estimate the error contributions from collocation and MOR separately. Both adjoints can be computed efficiently, if sparse direct solution methods are used for the linear systems. The resulting algorithm only requires a target accuracy and automatically generates and interpolates local adaptive ROMs.
We will demonstrate the accuracy of the estimators and the efficiency of the method with several simple examples and examples from vibroacoustics. We also discuss the advantages and limitations of the method related to other available approaches, which are based on pole matching [3, 1].

References

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