Structure-preserving and adaptive reduced order models of conservative dynamical systems

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Conservative systems describe reversible processes with no dissipative effects in a broad variety of phenomena of scientific and industrial interest. Examples of such systems are the n-body problem, the Lotka-Volterra equation of population dynamics, wave-type problems (e.g. the KdV equation, the Schrödinger equation, the shallow water equations, ...), conservation laws in fluid dynamics, and kinetic plasma models. In this talk we consider parametric conservative problems by resorting to their (semi-discrete) formulation as Hamiltonian dynamical systems. The development of reduced order models of such systems is challenged by several factors: (i) failing to preserve the geometric structure encoding the physical properties of the dynamics, such as invariants of motion or symmetries, might lead to instabilities and unphysical behaviors of the resulting approximate solutions; (ii) the slowly decaying Kolmogorov n-width of transport-dominated and non-dissipative phenomena demands large reduced spaces to achieve sufficiently accurate approximations; and (iii) nonlinear operators require hyper-reduction techniques that preserve the gradient structure of the flow velocity. We will discuss how to address these aspects via a nonlinear model order reduction approach based on evolving low-dimensional surrogate models on a phase space that adapts in time while being endowed with the geometric structure of the full model. To deal with nonlinear operators we propose a gradient-preserving strategy that consists in a sparse decomposition of the Hamiltonian gradient projected into the reduced space, followed by a discrete empirical interpolation of the resulting nonlinear factors. We will demonstrate the efficiency of the proposed techniques on a set of numerical tests.

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