Deep learning for reduced order modeling

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Reduced order modeling (ROM) techniques, such as the reduced basis method, provide nowadays an essential toolbox for the efficient approximation of parametrized differential problems, whenever they must be solved either in real-time, or in several different scenarios. These tasks arise in several contexts like, e.g., uncertainty quantification, control and monitoring, as well as data assimilation, ultimately representing key aspects in view of designing predictive digital twins in engineering or medicine. On the other hand, in the last decade deep learning algorithms have witnessed a dramatic blossoming in several fields, ranging from image and signal processing to predictive data-driven models. More recently, deep neural networks have also been exploited for the numerical approximation of differential problems yielding powerful physics-informed surrogate models.

In this talk we will explore different contexts in which deep neural networks can enhance the efficiency of ROM techniques, ultimately allowing the real-time simulation of large-scale nonlinear time-dependent problems. We show how to exploit deep neural networks (and a set of FOM snapshots) to build ROMs for parametrized PDEs in a fully non-intrusive way [3, 2], exploiting deep neural networks as main building block, ultimately yielding deep learning-based ROMs (DL-ROMs) and their further extension [4] to POD-enhanced DL-ROMs (POD-DL-ROMs). Moreover, we show a novel strategy for learning nonlinear ROM operators using deep neural networks [1], thus yielding hyper-reduced order models enhanced by deep neural networks (Deep-HyROMnets), where operator approximation is much more efficient in a projection-based ROM can be performed in an extremely efficient, yet accurate, way. Furthermore, we will also show how to improve a low-fidelity ROM through a multi-fidelity neural network regression technique that allows to merge low- and high-fidelity data, to enhance the ROM accuracy for the sake of input/output evaluations [5].
Through a set of applications from engineering including, e.g., structural mechanics and fluid dynamics problems, we will highlight the opportunities provided by deep learning in the context of ROMs for parametrized PDEs, as well as those challenges that are still open.

References

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